

# **Inference and Intraday Analysis of Diversified World Stock Indices**

by

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# Certificate

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of the requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged in the thesis. In addition I certify that all information sources and literature used are indicated in the thesis.

(Signed) 

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# Abstract

The benchmark framework provides an alternative paradigm for financial market modelling. Firstly, approaches to parameter estimation of discretely observed diffusion processes are examined, with particular emphasis on estimating function techniques. A new estimating function technique, called the transform function method, is introduced and applied to a class of stochastic differential equations. The advantage of the transform function method is that explicit information is not required about conditional moments and the existence of stationary transition densities. Despite the flexibility of the transform function technique, it suffers the same drawbacks as existing estimation methods with respect to drift estimation of financial data. The observation period required to estimate the drift function is much greater than what is available in financial markets. Notwithstanding the lack of available data for drift estimation, the diffusion function can be reliably estimated by the transform function method from short periods of frequently observed data. This thesis highlights that the benchmark approach of Platen (2002), where only estimates for diffusion coefficients are required, resolves the issue of drift estimation from financial market data.

Secondly, we extend the benchmark approach to incorporate modelling aspects particular to intraday data. This requires the construction of high-frequency diversified portfolios to approximate the growth optimal portfolio, the central building block of the benchmark model. Three different high-frequency indices are considered and it is demonstrated that an index with proportions based on total world market capitalisation provides the best proxy for the growth optimal portfolio. The benchmark model is then extended to intraday data through the introduction of a market activity process. This process is shown to be readily observable and as such, properties of market activity can be characterised. The analysis includes the consideration of the high-frequency indices in US Dollars and a further twenty currency denominations. As such, we reveal the pairwise co-

variation structures between the currency denominations considered. It is shown that the intraday benchmark model, which has market activity as the main parameter of interest, provides a largely accurate intraday description of financial markets.